Notes on a Crystal Mixer Performance

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Summary—**A simple relation between optimum rf source impedance for minimum noise figure and the nominal conversion loss of a mixer is derived. This impedance is related to the input mismatch and its dependence on the type of IF amplifier input circuit is discussed. Relations between the crystal noise temperature and mixer noise temperature as a function of conversion loss are derived for different load conditions at the image frequency terminals.**

IT IS the intent of this paper to discuss in a simple fashion two special aspects of crystal mixer performance; i.e., optimum rf source impedance and the interdependence of crystal and mixer noise temperatures.

A recent paper by Strum which presents a complete mathematical theory of mixer operation considers these matters on the basis of a three-terminal pair equivalent circuit.[1] The mixer is taken as a time-varying conductance (driven by a local oscillator) and this conductance is expanded in a Fourier series using the dc voltage-current characteristic. By considering the effects on this conductance of voltages at the signal, image, and intermediate frequencies respectively and comparing the resulting equations to those of a three-terminal pair network, an equivalent circuit is derived by analogy. The parameters of this circuit are related to the dc crystal characteristics via the Fourier coefficients.

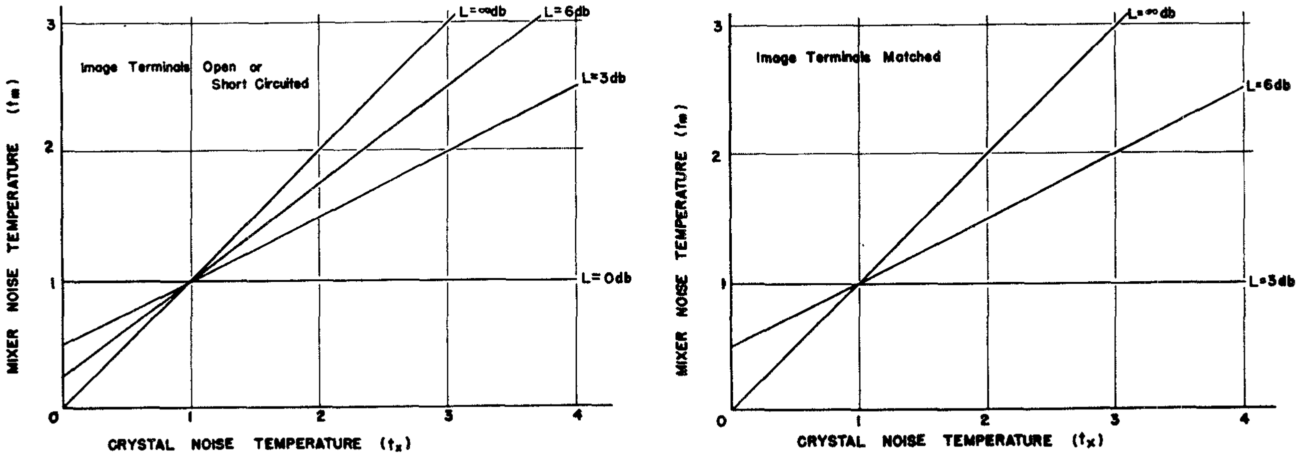

Fig. 1—Mixer noise temperature vs. crystal noise temperature.

Conventional network theory then permits the computation of an optimum source impedance in terms of the equivalent circuit parameters. Conversion loss can also be calculated in terms of these parameters, and finally optimum source impedance is plotted vs. conversion loss. In this paper we will arrive at the same result without the use of a detailed equivalent circuit, by proceeding as follows.

The mixer is considered as a two-terminal pair linear network with input terminals at the signal frequency and output terminals at the intermediate frequency. This representation is shown in Fig. 1.

The terminal voltages and currents are linearly related to each other by the general circuit parameters A, B, C, and D. These parameters include the effects of sum and image frequency terminations on the mixer, since terminals are not shown separately at these frequencies. Eqs. (1) and (2) state these relations.

$$\begin{aligned} e_1 &= Ae_2 + Bi_2 \\ i_1 &= Ce_2 + Di_2 \end{aligned} \tag{1}$$

$$\begin{aligned} e_2 &= \frac{De_1 - Bi_1}{AD - BC} \\ i_2 &= \frac{-Ce_1 + Ai_1}{AD - BC}. \end{aligned} \tag{2}$$

The relation $AD-BC=1$ is not used here since it is a consequence of the reciprocity theorem which is not necessarily assumed for this derivation. In order to achieve minimum noise figure it is necessary to have minimum conversion loss. This conversion loss must be defined on an "available power" basis[2] as the ratio of power available at the signal terminals to power available at the IF terminals. Note that it is not a function

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[1] P. D. Strum, "Some aspects of crystal mixer performance," Proc. IRE vol. 41, pp. 876–889; July, 1953.

[2] H. T. Friis, "Noise figure of radio receivers," Proc. IRE vol. 32, pp. 419–422; July, 1944.

of the IF input impedance. The available input power P_g and the available output power P_0 are given by

$$P_g = e^2/4R_g, \qquad P_0 = e_2^2/4Z_0, \tag{3}$$

where Z_0 is the output impedance of the network and e_2 is the open-circuit output voltage. e_2 is related to e_1 by (1) (with $i_2=0$), and e_1 is related to e using the input impedance of the network Z_i under open-circuit output conditions. After routine manipulating, we can write for the conversion loss L

$$L = A^2\left(\frac{Zi + R_g}{Zi}\right)^2 Z_0/R_g,$$

where

$$Zi = A/C, \qquad Z_0 = \frac{B + DR_g}{A + CR_g}\cdot \tag{4}$$

We minimize L in the usual fashion by setting $dL/dR_g=0$, and find that

$$R_{g0} = \sqrt{AB/CD}. \tag{5}$$

We still have not assumed the reciprocity theorem to show the existence of this optimum source impedance. However, to relate it simply to the optimum conversion loss it is necessary to assume that $AD-BC=1$. Substituting (5) in (4) yields

$$L_0 = AD(1 + \sqrt{(AD-1)/AD})^2. \tag{6}$$

L_0 is the optimum conversion loss and dependent on the crystal parameters only. It is not related to the circuits connected to the mixer.

The operating input impedance of the mixer will, of course, depend on the IF input impedance and will not in general be equal to the optimum source impedance. There will thus be an input mismatch. We calculate this mismatch for the two important cases of IF input short circuited (double tuned circuit) and IF input open circuited (single tuned circuit). Z_{in} is the input impedance to the mixer when loaded by an IF amplifier. We define the mismatch ρ by

$$\rho = R_{g0}/Z_{in}. \tag{7}$$

Case 1—IF input open circuited $Z_{in}=A/C$

$$\rho = \frac{\sqrt{AB/CD}}{A/C}\cdot$$

Applying (6) and the relation $AD-BC=1$, this can be shown equivalent to

$$\rho = \frac{L_0 - 1}{L_0 + 1}\cdot \tag{8}$$

Case 2—IF input short circuited $Z_{in}=B/D$

$$\rho = \frac{L_0 + 1}{L_0 - 1} \tag{9}$$

is demonstrated in a similar fashion.

L_0 is the optimum conversion loss and, for use in (8) and (9), the manufacturer's figure[3] can be used in the matched image case. In the open or shorted image terminal cases the actual conversion loss is different from the published values and should be calculated using the methods outlined in Strum's paper.

As a practical matter the desired source impedance is achieved by mismatching the source by an amount ρ as calculated using (8) or (9). The phase of this mismatch relative to the mixer input terminals depends on the type of IF input circuit. If the mismatch used is that of a tr tube or preselector at resonance and the characteristics of this tube are also used to achieve the "image open-circuit" condition, then careful consideration must be given to the phase characteristics of this tr tube to insure that the two conditions are compatible. To elaborate on this point we note that (9) demands that ρ be greater than unity if we are using a double tuned IF input circuit (with an impedance zero at resonance). This means that the generator source impedance shall be greater than the mixer input impedance. If the residual mismatch at resonance of a tr tube or filter is used to produce the required standing wave, its voltage minimum wants to be one-quarter wavelength from the mixer reference terminals. The same quarter wavelength is desirable at the image frequency in order to achieve the image open circuit condition necessary for minimum conversion loss. The two conditions are compatible only as long as there is no shift in phase of the tr reference planes such as would take place with an over-coupled cavity.

The important point here is that, depending on the phase characteristics of the tr tube or filter and the type of IF input circuit, it is possible for the residual source mismatch to be phased optimally incorrectly and thus to reduce the apparent improvement in noise figure from correct image conversion.

The noise temperature t_x of the crystal can be considered as the ratio to room temperature of the effective temperature of the resistors in the equivalent circuit. This ratio is greater than unity because of the active nature of a crystal rectifier which causes it to generate more noise than an equivalent resistor.

It is possible to proceed by calculating the noise voltage from each resistor in the equivalent circuit considered as a generator of voltage $\sqrt{4kt_xT_0BR}$. The contributions from all the resistors in the circuit are calculated at the output terminals and added on an RMS basis. All the resistors have a temperature t_xT_0 except the rf source impedance, which has a temperature T_0 (room temperature). The total noise voltage and the output impedance are then used to calculate an equivalent noise ratio t_m for the composite mixer-source system. This calculation is a tedious one, and expressing the results as a function of conversion loss is even more

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[3] H. Torrey and C. Whitmer, Radiation Laboratory Series, vol. 15, "Crystal Rectifiers," McGraw-Hill Book Co., Inc.; 1948.

complicated. We can proceed in a much simpler fashion by considering Fig. 2, which represents the mixer when the image terminals are either short or open circuited.

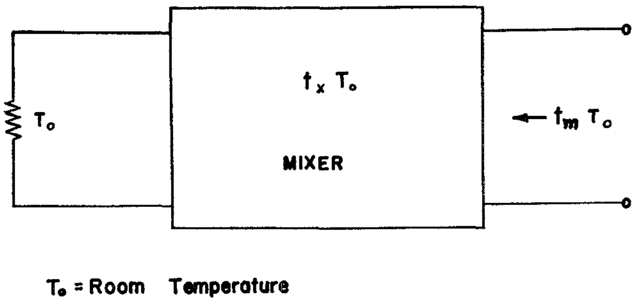

Fig. 2—Mixer as an "excess noise" generator.

If the whole system were at a temperature T_0, then the noise output would be given by

$$N_0 = KT_0B,$$

$$N_0 = \frac{KT_0B}{L} + kT_0B(1 - 1/L). \tag{10}$$

Writing (10) in this fashion permits us to identify the first term as representing the attenuated input noise and the second term as the excess noise generated in the mixer. If the mixer is actually at a higher temperature t_xT_0, then (10) can be rewritten as

$$N_0 = KT_0B/L + kt_xT_0B(1 - 1/L). \tag{11}$$

But, by definition, $N_0 = kt_mT_0B$, where t_m is the mixer noise ratio. Equating this expression to that in (11) and solving for t_m yields

$$t_m = \frac{1}{L}\left[t_x(L - 1) + 1\right]. \tag{12}$$

The case in which the image terminals are matched can be handled in the same manner. In this case there are two loads on the mixer at room temperature, or, what amounts to the same thing, the mixer accepts noise from the source in double the bandwidth. If everything were at room temperature,

$$N_0 = 2KT_0B,$$

$$N_0 = \frac{4KT_0B}{L} + 2KT_0B(1 - 2/L).$$

If the crystal is at temperature t_xT_0 and the mixer at an equivalent temperature t_mT_0 we can write,

$$\begin{aligned} 2Kt_mT_0B &= \frac{4KT_0B}{L} + 2Kt_xT_0B(1 - 2/L) \\ t_m &= \frac{2}{L}\left[t_x\left(\frac{L}{2} - 1\right) + 1\right] \end{aligned} \tag{13}$$

Eqs. (12) and (13) are plotted in Fig. 3. These curves are particularly significant, since they enable one to calculate the operating mixer noise temperature from the manufacturer's data. Essentially the manufacturer measures t_m under the second, or matched image condition. Using his values of t_m and conversion loss, Fig. 3 is used to determine t_x. Then using the actual operating conversion loss under the appropriate image terminal conditions (as calculated using Strum's method), the final value of t_m is found, again using Fig. 3. It is this value of t_m that is used in calculating the receiver noise figure from the well-known relation shown in (14),

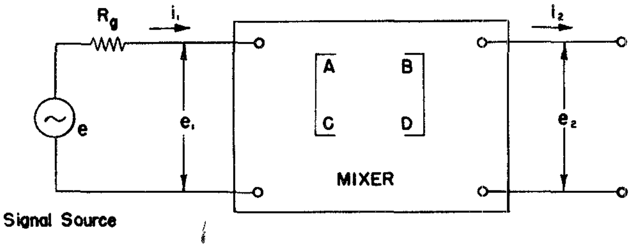

Fig. 3—Mixer equivalent circuit.

$$F = L(F_{\text{if}} + t_m - 1). \tag{14}$$

CONCLUSION

An expression of the optimum rf source impedance of a crystal mixer has been derived and related to the input standing wave ratio and conversion loss for different types of IF input circuits. The noise temperature of the mixer has been related to the crystal temperature as a function of conversion loss for matched, open-circuited, or short-circuited image frequency terminals.

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